4-1 Introduction to Matrices

Main Ideas

- Organize data in matrices.
- Solve equations involving matrices.

New Vocabulary

matrix
element
dimension
row matrix
column matrix
square matrix
zero matrix
equal matrices

Reading Math

Matrices The plural of *matrix* is *matrices*.

GET READY for the Lesson

There are many types of sport-utility vehicles (SUVs) in many prices and styles. So, Oleta makes a list of qualities to consider for some top-rated models. She organizes the information in a matrix to easily compare the features of each vehicle.

	Base Price ($)	Horse-power	Exterior Length (in.)	Cargo Space (ft^3)	Fuel Economy (mpg)
Hybrid SUV	19,940	153	174.9	66.3	22
Standard SUV	31,710	275	208.4	108.8	15
Mid-Size SUV	27,350	255	188.0	90.3	17
Compact SUV	21,295	165	175.2	64.1	21

Source: cars.com

Organize Data A **matrix** is a rectangular array of variables or constants in horizonal rows and vertical columns, usually enclosed in brackets.

Real-World EXAMPLE Organize Data into a Matrix

1 **The prices for two cable companies are listed below. Use a matrix to organize the information. When is each company's service less expensive?**

Metro Cable	
Basic Service (26 channels)	$11.95
Standard Service (53 channels)	$30.75
Premium Channels (in addition to Standard Service)	
• One Premium	$10.00
• Two Premiums	$19.00
• Three Premiums	$25.00

Cable City	
Basic Service (26 channels)	$9.95
Standard Service (53 channels)	$31.95
Premium Channels (in addition to Standard Service)	
• One Premium	$8.95
• Two Premiums	$16.95
• Three Premiums	$22.95

Organize the costs into labeled columns and rows.

	Basic	Standard	Standard Plus One Premium	Standard Plus Two Premiums	Standard Plus Three Premiums
Metro Cable	11.95	30.75	40.75	49.75	55.75
Cable City	9.95	31.95	40.90	48.90	54.90

Metro Cable has the best price for standard service and standard plus one premium channel. Cable City has the best price for the other categories.

CHECK Your Progress

1. Use a matrix to organize and compare the following information about some roller coasters.

Roller Coaster	Batman the Escape	Great White	Mr. Freeze
Speed (mph)	55	50	70
Height (feet)	90	108	218
Length (feet)	2300	2562	1300

Reading Math

Element The elements of a matrix can be represented using double subscript notation. The element a_{ij} is the element in row i column j.

In a matrix, numbers or data are organized so that each position in the matrix has a purpose. Each value in the matrix is called an **element**. A matrix is usually named using an uppercase letter.

$$A = \begin{bmatrix} 2 & 6 & 1 \\ 7 & 1 & 5 \\ 9 & 3 & 0 \\ 12 & 15 & 26 \end{bmatrix}$$

4 rows

3 columns

The element 15 is in row 4, column 2.

A matrix can be described by its **dimensions**. A matrix with m rows and n columns is an $m \times n$ matrix (read "m by n"). Matrix A above is a 4×3 matrix since it has 4 rows and 3 columns.

EXAMPLE 2 Dimensions of a Matrix

State the dimensions of matrix B if $B = \begin{bmatrix} 1 & -3 \\ -5 & 18 \\ 0 & -2 \end{bmatrix}$.

$$B = \begin{bmatrix} 1 & -3 \\ -5 & 18 \\ 0 & -2 \end{bmatrix}$$

3 rows

2 columns

Since matrix B has 3 rows and 2 columns, the dimensions of matrix B are 3×2.

CHECK Your Progress

2. State the dimensions of matrix L if $L = \begin{bmatrix} -2 & 1 & 3 & -4 \\ 0 & 3 & 0 & 7 \end{bmatrix}$.

Certain matrices have special names. A matrix that has only one row is called a **row matrix**, while a matrix that has only one column is called a **column matrix**. A matrix that has the same number of rows and columns is called a **square matrix**. Another special type of matrix is the **zero matrix**, in which every element is 0. The zero matrix can have any dimension.

Equations Involving Matrices Two matrices are considered **equal matrices** if they have the same dimensions and if each element of one matrix is equal to the corresponding element of the other matrix.

Example: $\begin{bmatrix} 5 & 6 & 0 \\ 0 & 7 & 2 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 0 \\ 0 & 7 & 2 \\ 3 & 1 & 4 \end{bmatrix}$ The matrices have the same dimensions and the corresponding elements are equal. The matrices are equal.

Non-example: $\begin{bmatrix} 6 & 3 \\ 0 & 9 \\ 1 & 3 \end{bmatrix} \neq \begin{bmatrix} 6 & 0 & 1 \\ 3 & 9 & 3 \end{bmatrix}$ The matrices have different dimensions. They are not equal.

Non-example: $\begin{bmatrix} 1 & 2 \\ 8 & 5 \end{bmatrix} \neq \begin{bmatrix} 1 & 8 \\ 2 & 5 \end{bmatrix}$ Not all corresponding elements are equal. The matrices are not equal.

The definition of equal matrices can be used to find values when elements of equal matrices are algebraic expressions.

EXAMPLE Solve an Equation Involving Matrices

3 **Solve** $\begin{bmatrix} y \\ 3x \end{bmatrix} = \begin{bmatrix} 6 - 2x \\ 31 + 4y \end{bmatrix}$ **for** x **and** y.

Since the matrices are equal, the corresponding elements are equal. When you write the sentences to show this equality, two linear equations are formed.

$y = 6 - 2x$

$3x = 31 + 4y$

This system can be solved using substitution.

$3x = 31 + 4y$	Second equation
$3x = 31 + 4(6 - 2x)$	Substitute $6 - 2x$ for y.
$3x = 31 + 24 - 8x$	Distributive Property
$11x = 55$	Add $8x$ to each side.
$x = 5$	Divide each side by 11.

To find the value for y, substitute 5 for x in either equation.

$y = 6 - 2x$	First equation
$y = 6 - 2(5)$	Substitute 5 for x.
$y = -4$	Simplify.

The solution is (5, −4).

CHECK Your Progress

3. Solve $\begin{bmatrix} 5x + 2 & y - 4 \\ 0 & 4z + 6 \end{bmatrix} = \begin{bmatrix} 12 & -8 \\ 0 & 2 \end{bmatrix}$.

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CHECK Your Understanding

Example 1 (pp. 162-163)

WEATHER For Exercises 1 and 2, use the table that shows a five-day forecast indicating high (H) and low (L) temperatures.

Fri	Sat	Sun	Mon	Tue
H 88	H 88	H 90	H 86	H 85
L 54	L 54	L 56	L 53	L 52

1. Organize the temperatures in a matrix.

2. Which day will be the warmest?

Example 2 (p. 163)

State the dimensions of each matrix.

3. $\begin{bmatrix} 3 & 4 & 5 & 6 & 7 \end{bmatrix}$

4. $\begin{bmatrix} 10 & -6 & 18 & 0 \\ -7 & 5 & 2 & 4 \\ 3 & 11 & 9 & 7 \end{bmatrix}$

Example 3 (p. 164)

Solve each equation.

5. $\begin{bmatrix} x+4 \\ 2y \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \end{bmatrix}$

6. $\begin{bmatrix} 9 & 13 \end{bmatrix} = \begin{bmatrix} x+2y & 4x+1 \end{bmatrix}$

Exercises

HOMEWORK HELP

For Exercises	See Examples
7–8	1
9–14	2
15–20	3

Organize the information in a matrix.

7.

Ocean	Area (mi^2)	Average Depth (ft)
Pacific	60,060,700	13,215
Atlantic	29,637,900	12,880
Indian	26,469,500	13,002
Southern	7,848,300	16,400
Arctic	5,427,000	3,953

Source: factmonster.com

8.

Top Hockey Goalies				
Goalie	Games	Wins	Losses	Ties
Roy	1029	551	315	131
Sawchuk	971	447	330	172
Plante	837	435	247	146
Esposito	886	423	306	152
Hall	906	407	326	163

Source: factmonster.com

State the dimensions of each matrix.

9. $\begin{bmatrix} 6 & -1 & 5 \\ -2 & 3 & -4 \end{bmatrix}$

10. $\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$

11. $\begin{bmatrix} 0 & 0 & 8 \\ 6 & 2 & 4 \\ 1 & 3 & 6 \\ 5 & 9 & 2 \end{bmatrix}$

12. $\begin{bmatrix} -3 & 17 & -22 \\ 9 & 31 & 16 \\ 20 & -15 & 4 \end{bmatrix}$

13. $\begin{bmatrix} 17 & -2 & 8 & -9 & 6 \\ 5 & 11 & 20 & -1 & 4 \end{bmatrix}$

14. $\begin{bmatrix} 16 & 8 \\ 10 & 5 \\ 0 & 0 \end{bmatrix}$

Solve each equation.

15. $\begin{bmatrix} 4x & 3y \end{bmatrix} = \begin{bmatrix} 12 & -1 \end{bmatrix}$

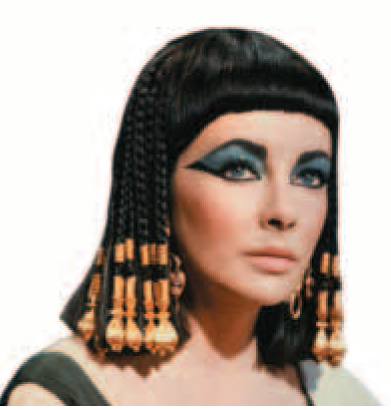
16. $\begin{bmatrix} 2x & 3 & 3z \end{bmatrix} = \begin{bmatrix} 5 & 3y & 9 \end{bmatrix}$

17. $\begin{bmatrix} 4x \\ 5 \end{bmatrix} = \begin{bmatrix} 15+x \\ 2y-1 \end{bmatrix}$

18. $\begin{bmatrix} x+3y \\ 3x+y \end{bmatrix} = \begin{bmatrix} -13 \\ 1 \end{bmatrix}$

19. $\begin{bmatrix} 2x+y \\ x-3y \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \end{bmatrix}$

20. $\begin{bmatrix} 4x-3 & 3y \\ 7 & 13 \end{bmatrix} = \begin{bmatrix} 9 & -15 \\ 7 & 2z+1 \end{bmatrix}$



DINING OUT **For Exercises 21 and 22, use the following information.** A newspaper rated several restaurants by cost, level of service, atmosphere, and location using a scale of ★ being low and ★★★★ being high.

Restaurant	Cost	Service	Atmosphere	Location
Catalina Grill	★★	★	★	★
Oyster Club	★★★	★★	★	★★
Casa di Pasta	★★★★	★★★	★★★	★★★
Mason's Steakhouse	★★	★★★★	★★★★	★★★

Real-World Link

Adjusting for inflation, *Cleopatra* (1963) is the most expensive movie ever made. Its $44 million budget is equivalent to $306,867,120 today.

Source: *The Guiness Book of Records*

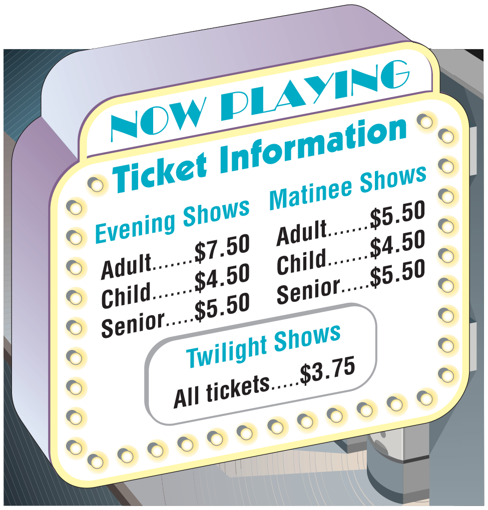
21. Write a 4 × 4 matrix to organize this information.

22. Which restaurant would you select based on this information, and why?

MOVIES **For Exercises 23 and 24, use the advertisement shown at the right.**

23. Write a matrix for the prices of movie tickets for adults, children, and seniors.

24. What are the dimensions of the matrix?



HOTELS **For Exercises 25 and 26, use the costs for an overnight stay at a hotel that are given below.**

Single Room: $60 weekday; $79 weekend

Double Room: $70 weekday; $89 weekend

Suite: $75 weekday; $95 weekend

25. Write a 3 × 2 matrix that represents the cost of each room.

26. Write a 2 × 3 matrix that represents the cost of each room.

EXTRA PRACTICE See pages 897, 929.

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H.O.T. Problems

27. **RESEARCH** Use the Internet or other resource to find the meaning of the word *matrix*. How does the meaning of this word in other fields compare to its mathematical meaning?

28. **OPEN ENDED** Give examples of a row matrix, a column matrix, a square matrix, and a zero matrix. State the dimensions of each matrix.

CHALLENGE **For Exercises 29 and 30, use the matrix at the right.**

$$\begin{bmatrix} 1 & 3 & 6 & 10 & 15 & \cdots \\ 2 & 5 & 9 & 14 & 20 & \cdots \\ 4 & 8 & 13 & 19 & 26 & \cdots \\ 7 & 12 & 18 & 25 & 33 & \cdots \\ 11 & 17 & 24 & 32 & 41 & \cdots \\ 16 & 23 & 31 & 40 & 50 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

29. Study the pattern of numbers. Complete the matrix for column 6 and row 7.

30. In which row and column will 100 occur?

31. ***Writing in Math*** Use the information about SUVs on page 162 to explain how a matrix can help Sabrina decide which SUV to buy.

Bettman/CORBIS

STANDARDIZED TEST PRACTICE

32. ACT/SAT The results of a recent poll are organized in the matrix.

$$\begin{array}{l} \\ \text{Proposition 1} \\ \text{Proposition 2} \\ \text{Proposition 3} \end{array} \begin{array}{c} \text{For} \quad \text{Against} \\ \begin{bmatrix} 1553 & 771 \\ 689 & 1633 \\ 2088 & 229 \end{bmatrix} \end{array}$$

Based on these results, which conclusion is NOT valid?

A There were 771 votes cast against Proposition 1.

B More people voted against Proposition 1 than voted for Proposition 2.

C Proposition 2 has little chance of passing.

D More people voted for Proposition 1 than for Proposition 3.

33. REVIEW The chart shows an expression evaluated for four different values of x.

x	$x^2 + x + 1$
1	3
2	7
3	13
5	31

A student concludes that for all values of x, $x^2 + x + 1$ produces a prime number. Which value of x serves as a counterexample to prove this conclusion false?

F −4 **H** −2

G −3 **J** 4

Spiral Review

Solve each system of equations. (Lesson 3-5)

34. $3x - 3y = 6$
$-6y = -30$
$5z - 2x = 6$

35. $3a + 2b = 27$
$5a - 7b + c = 5$
$-2a + 10b + 5c = -29$

36. $3r - 15s + 4t = -57$
$9r + 45s - t = 26$
$-6r + 10s + 3t = -19$

37. BUSINESS A factory is making skirts and dresses from the same fabric. Each skirt requires 1 hour of cutting and 1 hour of sewing. Each dress requires 2 hours of cutting and 3 hours of sewing. The cutting department can cut up to 120 hours each week and the sewing department can sew up to 150 hours each week. If profits are \$12 for each skirt and \$18 for each dress, how many of each should the factory make for maximum profit? (Lesson 3-4)

38. Write an equation in slope-intercept form of the line that passes through the points indicated in the table. (Lesson 2–4)

39. Write an equation in standard form of the line that passes through the points indicated in the table. (Lesson 2–1)

x	y
−3	−1
2	$\frac{7}{3}$
3	3

Find each value if $f(x) = x^2 - 3x + 2$. (Lesson 2-1)

40. $f(3)$ **41.** $f(0)$ **42.** $f(2)$ **43.** $f(-3)$

GET READY for the Next Lesson

Find the value of each expression. (Lesson 1-2)

44. $8 + (-5)$ **45.** $6(-3)$ **46.** $\frac{1}{2}(34)$ **47.** $-5(3 - 18)$